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QUANTICA CAPITAL

QUARTERLY INSIGHTS

THE VALUE OF DIVERSIFICATION IN TREND-FOLLOWING

Why Diversification is so much more important in Trend-Following than in Long-Only Strategies

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Executive summary

'Diversification is the only free lunch in Finance'

This famous quote by Nobel laureate Harry Markowitz from 1952 revolutionized the investment industry and has lost none of its importance and relevance to this day.

According to economic theory, investors are only compensated for the non-diversifiable, systematic 'market' risk, but not for any individual, market specific or active portfolio risks.

The resounding success of passive investment strategies in the recent decade, e.g. passive ETF vehicles, compared to active strategies seems to confirm the theory in an impressive way.

But how many positions are needed in practice to take advantage of the diversification potential embedded in a specific investment strategy?

At Quantica, we have recently expanded our investment universe for the Quantica Managed Futures Program (QMF) from 64 to 84 of the most liquid, global futures markets. Quantifying the diversification benefit of this expansion is, of course, one of the important tasks we had to carefully complete prior to implementing such enhancement to the strategy.

In this Quarterly Insights we take a closer look at the determinants of portfolio diversification and present a simple but theoretically sound model to quantify the diversification potential for different investment strategies, with an emphasis on systematic trend-following.

Starting with a simple model, we demonstrate how the diversification gain when expanding a portfolio's investment universe is linked to the average cross-correlation of the underlying return streams and the portfolio size.

We show that the diversification potential is limited, and essentially inversely proportional to the square root of the average cross-correlation of the underlying return streams.

The non-linear relationship between a portfolio's underlying average cross-correlation and the resulting portfolio Sharpe ratio leads to significant differences between different investment opportunities and strategies.

As an important takeaway, we show that the diversification potential of trend-following strategies is significantly higher than e.g. for equity long only or risk parity strategies. For pure stock portfolios, the diversification potential is no longer significant for portfolios with more than 15-20 positions, and the Sharpe ratio cannot be increased much more through pure diversification.

Notably, for trend-following strategies applied to a universe of liquid futures markets, this *diversification threshold* is significantly higher. Trend-following portfolios with 100 positions (for generic trend-following models) and even 200 positions (applying Quantica's risk-adjusted, relative trend-following methodology) could still benefit from a sizable diversification gain when considering a further investment universe expansion.

We demonstrate how a careful evaluation and integration of diversification considerations into the investment process can be crucial to the long-term success of a trend-following strategy. As a result, the Portfolio Sharpe ratio can be increased up to a well quantifiable level, and without compromising style consistency or other desirable features of the trend-following strategy.

Introduction

As the saying goes, it was common practice in the 19th century in certain saloons in the US to attract guests with so-called 'free lunches'. The flip side of the coin was usually over-salted food that tempted customers to consume more of the overpriced drinks.

Translated to the modern world of investing, this means that any kind of excess return over "risk-free rates" - which are essentially negative these days - must be accompanied, or paid for, by increased risk in some form or another. In simple words: **"There ain't such a thing as a free lunch"** .

Frequently quoted in the 20th century, this saying formed the simple basis for some groundbreaking theories in Modern Finance, based on the "No-Arbitrage Pricing Theory" founded, amongst others, by Nobel laureates Robert C. Merton and Myron Scholes.

In 1952, another Nobel Prize winner, Harry Markowitz put this statement into perspective with his infamous saying "**Diversification is the only free lunch in Finance**". And in fact, his Capital Asset Pricing Model (CAPM) was the foundation of Modern Portfolio Theory, which essentially concludes that investors are not compensated for any market specific risk, but only for systematic, non-diversifiable risks. Or, in other words, the higher the diversification, the better.

Advocates of a more active investment style, on the other hand, would argue that over-diversification will always disappoint, as it will generate average returns, hence preventing any possibility of outperformance, which seems against the nature of most investors.

Therefore, it is only natural for any investor to ask how many positions they should optimally have in their portfolio, to exploit the obvious benefits of diversification without being overdiversified, or paying any other price related to such expansion. Any increase in the number of portfolio constituents usually comes with a price, be it in the form of increased transaction costs, or simply in the form of increased complexity. Therefore, a sound understanding of the potential benefit of extending the portfolio size, or the investment universe, is key.

At Quantica, we apply a fully systematic investment process to the most liquid global exchange traded futures markets. The center of our investment philosophy is the conviction that attractive long-term risk-adjusted returns can be generated by implementing a fully systematic trend-following approach, that is based on the identification of risk-adjusted, relative out- or underperformance of individual markets against each other. We believe that such positive 'trend-following' risk premia can be harvested in any individual market over a sufficiently long period.

We are agnostic on which markets will generate superior long-term risk-adjusted trend-following returns compared to others, and we do believe all markets have eventually the same expected long-term risk-adjusted trend-following profit potential. Hence, diversification is key to our goal to generate the best possible long-term risk-adjusted returns and to offer maximum liquidity and capacity.

On the other hand, the universe of accessible and liquid futures markets is limited, and the challenge is to factor in the increasing transaction cost of adding less liquid markets against the benefit of increased diversification.

Therefore, having a model to value the diversification benefit of adding new markets and extending the investment universe, is important to us.

In this note, we will present a simple but theoretically sound model that expresses the risk-adjusted return – or Sharpe ratio – of a portfolio as a simple function of the number of portfolio constituents, its average cross-correlation of the constituents' return streams, and the average Sharpe ratio of its constituents.

In fact, based on the average cross-correlation of the underlying returns, we can easily compute a diversification multiplier that expresses the theoretical upside potential to increase the portfolio Sharpe ratio simply through adding more instruments, or put differently, purely through diversification.

This simple formula allows us to precisely quantify the expected increase in Sharpe ratio by increasing the number of portfolio constituents with specific return and correlation characteristics.

We apply our methodology to analyze the diversification potential for different investment opportunities and strategies, including single name equity and global futures markets. We show results for long-only and risk parity strategies as well as generic trend-following strategies and more specifically Quantica's risk-adjusted, relative trend-following strategy implemented in the Quantica Managed Futures Program (QMF).

A simple formula to measure the diversification benefit of a diversified portfolio

In order to obtain a quantitative understanding of the benefit of diversification, we first have to settle on a measure of portfolio performance that isn't dependent solely on the constituents' expected returns. To that end, we choose to look at the Sharpe ratio – or risk-adjusted return – which famously also takes a portfolio's risk into account and which, for our purposes, we define for a portfolio with (log-)return R in the slightly simplified form

$$\mathrm{Sharpe}(R) = \frac{E[R]}{\sqrt{\mathrm{Var}(R)}}.$$

Here $E[R]$ denotes the annualized expected return of the portfolio (log-)return R and $\mathrm{Var}(R)$ denotes its variance – a measure of the portfolio's variability or risk. Note that our definition of the Sharpe ratio agrees with the original one under the assumption of a zero risk-free rate.

To understand the dependence of the Sharpe ratio on diversification, we start with a review of the variability, or risk, of an equally weighted portfolio of n correlated (log-)return streams $R_1, \dots, R_n$ with expected returns $\mu_1, \dots, \mu_n$, equal volatilities σ (i.e. the annualized square root of the variances), and correlation matrix ρ_{ij}. We show in the Appendix that the variance of such a portfolio can be expressed as a function of the number of constituents n, the individual volatilities σ, and the average cross-correlation $\bar{\rho} = \frac{1}{n^2-n}\sum_{i \neq j} \rho_{ij}$ by the simple formula

$$\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} R_i\right) = \sigma^2 \left(\bar{\rho} + \frac{1-\bar{\rho}}{n}\right).$$

It follows (a detailed derivation is shown in the Appendix) that the Sharpe ratio of such a portfolio is proportional to the average Sharpe ratio of its constituents $\bar{s} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Sharpe}(R_i)$, with a proportionality factor $m(\bar{\rho}, n)$ that only depends on the number of constituents and their average cross-correlation $\bar{\rho}$:

$$\mathrm{Sharpe}\left(\frac{1}{n}\sum_{i=1}^{n} R_i\right) = \bar{s} \cdot \frac{1}{\sqrt{\bar{\rho} + \frac{1-\bar{\rho}}{n}}} = \bar{s} \cdot m(\bar{\rho}, n)\,.$$

This gives us a simple quantitative description of the expected increase in Sharpe ratio as we add more return streams to a given portfolio. Notably, the *diversification multiplier* function $m(\bar{\rho}, n)$ depends only on the *average* cross-correlation $\bar{\rho}$ and the number of instruments. We do not need to assume that all cross-correlations are equal to $\bar{\rho}$, as is sometimes claimed in the relevant industry literature.

Furthermore, we can immediately observe a few interesting edge cases from the above formula:

- In the limit $n \to \infty$, with the number of constituents growing to infinity, the Sharpe ratio converges to $\bar{s} \cdot \frac{1}{\sqrt{\bar{\rho}}}$, i.e. the diversification multiplier is inversely proportional to the square root of the average cross-correlation.
- The above limit may not exist if $\bar{\rho}$ is zero or negative. In that case the formula is only defined for n up to $\frac{\bar{\rho}-1}{\bar{\rho}}$, which precisely coincides with the theoretical lower bound on the average cross-correlation in any universe of size n: $\bar{\rho} \geq -\frac{1}{n-1}$.
- If $\bar{\rho} = 0$, then the formula reduces to $\bar{s} \cdot \sqrt{n}$ and we see that the Sharpe ratio can grow without bounds, proportionally to the square root of the number of instruments.
- If instead all the constituents are perfectly correlated, i.e. $\bar{\rho} = 1$, (or if $n = 1$), the multiplier $m(\bar{\rho}, n) = 1$, and the portfolio Sharpe ratio is equal to $\bar{s}$, and there is no diversification benefit at all.

In conclusion, we have derived a simple, theoretically sound but powerful model for how the Sharpe ratio of an equal-weighted portfolio with constituents of equal volatility is linked to the *number of constituents*, their *average cross-correlation*, and their *average risk-adjusted return*.

Empirical analysis for different investment strategies

With the simple formula derived in the previous section, we can now move on to empirically compute the multiplier $m(\bar{\rho}, n)$ measuring the pure diversification benefit obtained from adding instruments to different portfolios just by knowing the average cross-correlation $\bar{\rho}$ of their constituents' returns.

Table 1 shows the empirical average cross-correlation of the (log-)return streams of long-only, risk-parity and generic trend-following strategies and the diversification multiplier, i.e. the diversification benefit for different portfolio sizes *n*. To highlight the striking differences for different asset classes and investment strategies, we apply the methodology to single name stock portfolios (represented by the S&P 500 universe), and diversified futures portfolios represented by 84 of the most liquid global futures markets currently traded within the Quantica Managed Futures (QMF) program. Finally, as a special case, we also evaluate the universe of 16 of the largest trend-following CTAs, which typically show very high cross-correlations, and hence limited diversification potential.

A first interesting observation is that a risk parity version of a long-only basket of stocks helps to

Strategy Description	Average Cross-Correlation	Diversification multiplier for i instruments						
		i = 1	10	25	50	84	100	∞
Equal-Weighted (S&P 500 constituents)	39%	1.0	1.5	1.6	1.6	1.6	1.6	1.6
Risk Parity (S&P 500 constituents)	34%	1.0	1.6	1.6	1.7	1.7	1.7	1.7
Generic Trend-Following (S&P 500 constituents)	12%	1.0	2.2	2.5	2.7	2.8	2.8	2.9
Risk Parity (broad futures universe*)	10%	1.0	2.3	2.7	2.9	3.0	3.0	3.2
Generic Trend-Following (broad futures universe*)	7.6%	1.0	2.4	3.0	3.3	3.4	3.4	3.6
QMF Program (broad futures universe*)	4.4%	1.0	2.7	3.5	4.0	4.2	4.3	4.8
Equal-Weighted (CTA Portfolio**)	65%	1.0	1.2	1.2	1.2	1.2	1.2	1.2
Risk Parity (CTA Portfolio**)	65%	1.0	1.2	1.2	1.2	1.2	1.2	1.2

Table 1: Average cross-correlation between instrument returns and diversification multipliers for a portfolio of different sizes. Data covers the period 2010-2021. Risk parity strategy chooses weight so that each constituent has an equal volatility target. *84 of the most liquid futures across equities, fixed income, commodities and FX. **16 largest CTAs as of 30 September 2021. Source: Quantica Capital.

decrease the average cross-correlation between instruments, and therefore increases the diversification potential of adding new instruments. This eventually leads to a higher achievable portfolio Sharpe ratio.

Secondly, a trend-following signal applied to the S&P 500 universe leads to lower average cross-correlations compared to passive long-only strategies. Therefore, the potential to increase the portfolio Sharpe ratio through diversification alone is much higher. This already points to greater importance of diversification in trend-following strategies.

Applied to the S&P 500 universe, the diversification multiplier of an equal weighted portfolio can reach a value of 1.6 (1.7 for its risk parity counterpart). For a generic trend-following model applied to the S&P 500 universe, the diversification multiplier can reach 2.9, and hence the value of diversification is 50% higher compared to the equally weighted strategies, which translates into a 50% higher expected Sharpe ratio multiplier.

The highest diversification multipliers are achieved in trend-following strategies applied to a diversified set of futures markets. The generic trend-following model applied to our universe of 84 futures markets reaches a diversification multiplier of 3.6, again indicating the importance of diversification in trend-following portfolios.

The empirical results for our Quantica Managed Futures (QMF) Program differ from a generic trend-following model approach by having an even lower average cross-correlation between its constituents return streams of 4.4% compared to 7.6% for a generic trend-following strategy. The lower cross-correlation brings the diversification multiplier to 4.2 for a portfolio of 84 assets, compared to 3.4 for a generic trend-following program.

In contrast, the diversification benefit is limited for CTA portfolios, which highlights the importance of fund selection in the CTA space. The relatively high average cross-correlation of 0.65 characterizing the universe of the largest CTAs leads to a limited diversification multiplier of around 1.2. Figure 1 illustrates the non-linear relationship between the average cross-correlation of the return streams and the diversification multiplier for different number of instruments. On this graph, we see how the diversification benefit gets more important as the

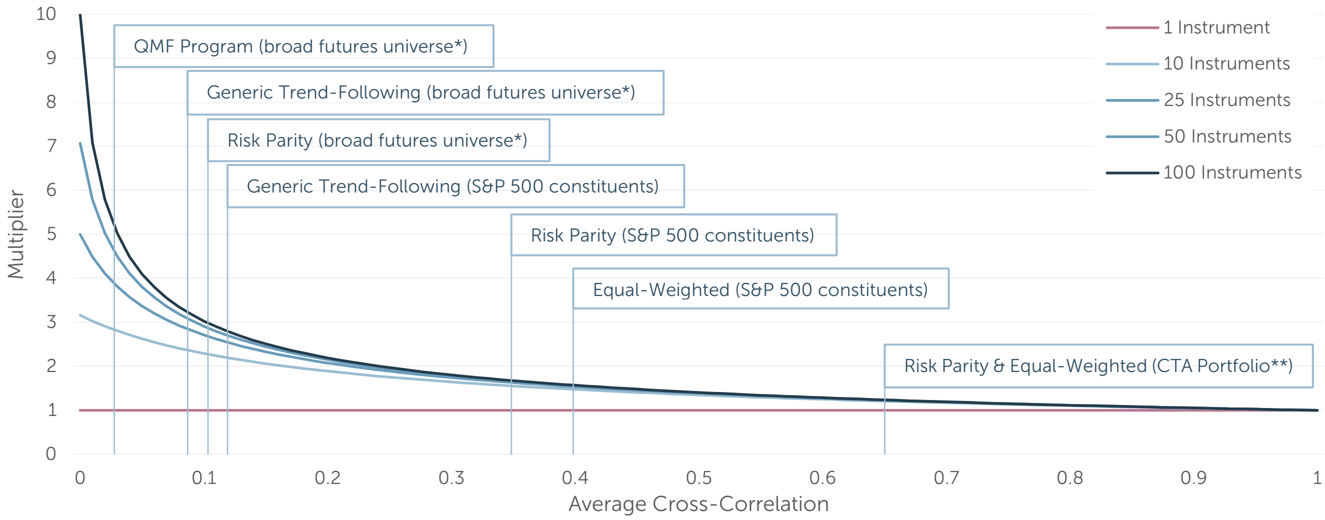

Figure 1: Multiplier as a function of average cross-correlation between return streams and diversification multipliers for a portfolio of different sizes. Data covers the period 2010-2021. Risk parity strategy chooses weight so that each constituent has an equal volatility target. *84 of the most liquid futures across equities, fixed income, commodities and FX. **16 largest CTAs as of 30 September 2021. Source: Quantica Capital.

average cross-correlation gets lower. Any signal or return stream added to a portfolio that helps reducing the average cross-correlation between instruments allows to move towards the left on the graph and therefore to climb up the diversification benefit curve. The plot also shows the more instruments there are in the portfolio, the steeper the diversification benefit curve, although the steepness only has a meaningful impact for sufficiently low average cross-correlations between return streams. The increasing non-linearity of the diversification multiplier at lower cross-correlations illustrates the importance of diversification for trend-following strategies such as the QMF program.

The question that arises from those results is

What is the limit of diversification?

Mathematically, the multiplier increases to the limit $\frac{1}{\sqrt{\bar{\rho}}}$ as the number of instruments gets higher and higher. However, in real life, the additional benefit of adding more positions is reduced by higher additional operational and trading costs. Furthermore, there is often only a limited set of cost-efficient and liquid instruments to choose from.

In the next section we introduce an intuitive diversification threshold that allows to compute an 'optimal' portfolio size that takes advantage of a 'sufficiently significant' portion of the diversification benefit.

How many positions are needed to take advantage of the diversification potential?

In the previous sections we have demonstrated why diversification – in the form of an increase in the number of portfolio constituents – is beneficial for any investment strategy, and how we can quantify the additional benefit of adding an additional return stream to a portfolio. We have shown that the diversification benefit is essentially a function of the average cross-correlation of the returns only, which depends heavily on the investment strategy under consideration. In fact, thanks to the simple methodology applied before, we can go one step further and precisely quantify how the diversification benefit is adding to a portfolio's Sharpe ratio, depending on the Sharpe ratio of the added return stream as well as it's cross-correlations to the already held ones. This is crucial information given the potentially added complexity in trading the additional instrument as well as the potentially higher trading costs incurred.

We have demonstrated that for given return-, risk- and correlation characteristics $\bar{s}$ and $\bar{\rho}$, the portfolio Sharpe ratio is limited by $\bar{s} \cdot \frac{1}{\sqrt{\bar{\rho}}}$. Hence, it is natural to ask for the smallest number N_p of constituents needed to reach a certain percentage $p \in [0,1]$ of this maximally achievable Sharpe ratio. This number turns out to be independent of $\bar{s}$ and is given by (for a technical derivation see the Appendix)

$$N_p = \frac{1-\bar{\rho}}{\bar{\rho}} \cdot \frac{p^2}{1-p^2}.$$

With that in hand, we can introduce $N_{95\%}$, the *95% diversification threshold*, representing the number of portfolio constituents needed to reach 95% of the maximally achievable Sharpe ratio for the given correlation characteristics. The lower this number is, the less it makes sense to try and improve the strategy purely through diversification. The 95% percentile is of course arbitrary and can be set differently, but it seems a reasonable choice for practical applications. Again, we emphasize that the diversification threshold depends only on the average cross-correlation of the underlying portfolio constituents. We do not need to assume that all cross-correlations are equal to $\bar{\rho}$.

	Fitted Results from Empirical Simulation				
Strategy Description	Average Sharpe-Ratio	Average Cross-Correlation	Maximum Multiplier	Max Sharpe-Ratio	95% Diversification Threshold
Equal-Weighted (S&P 500 constituents)	0.62	41%	1.6	1.0	**14**
Risk Parity (S&P 500 constituents)	0.71	34%	1.7	1.2	**19**
Generic Trend-Following (S&P 500 constituents)	0.24	10%	3.1	0.8	**86**
Risk Parity (broad futures universe*)	0.40	11%	3.1	1.2	**81**
Generic Trend-Following (broad futures universe*)	0.26	7.2%	3.7	1.0	**126**
QMF Program (broad futures universe*)	0.33	4.3%	4.8	1.6	**217**
Equal-Weighted (CTA Portfolio**)	0.39	69%	1.2	0.5	**4**
Risk Parity (CTA Portfolio**)	0.36	65%	1.2	0.4	**5**

Table 2: Empirically fitted results from a simulation of random portfolios of different sizes. Data covers the period 2010-2021. Risk parity strategy chooses weight so that each constituent has an equal volatility target. *84 of the most liquid futures across equities, fixed income, commodities and FX. **16 largest CTAs as of 30 September 2021. Source: Quantica Capital.

To compute the threshold and the maximal achievable Sharpe ratio for any strategy, we can estimate the average Sharpe ratios and the cross-correlations of its constituent return streams and use the simple diversification multiplier formula as outlined in the previous section.

Empirical validation of the Theoretical Diversification Model

To empirically validate the theoretical approach in practice, without making any assumptions on the portfolio composition, we apply a simulation technique to construct randomly selected portfolios of different sizes and compute the resulting average Sharpe ratios. We then fit our model to the resulting "expected Sharpe vs. number of constituents"-curve, to obtain alternative empirical estimates of the expected Sharpe ratios and the cross-correlations. Notably, this approach does not have to make any assumptions on the distribution of the portfolio's return stream, e.g. the equality or stationarity of volatilities.

We found that the empirical results impressively confirm the naïve approach from the diversification multiplier formula.

Table 2 summarizes the results of this empirical approach applied to the strategies considered in the previous section.

For the single name equity universe represented by the S&P 500 Index, we find a 95% diversification threshold of 14, indicating that a portfolio size of 14 randomly selected single stocks already takes advantage of 95% of the maximum diversification potential, resulting from the average cross-correlation of 0.41 for a passive long-only strategy. Portfolios consisting of 15-20 reasonably diversified single name stocks are already mainly exposed to systematic Equity risk, and the diversification benefit dries up quickly for larger portfolio sizes.

Risk parity strategies appear to benefit somewhat more from the increased diversification potential, which can increase the portfolio's achievable Sharpe ratios by up to 20%.

For *trend-following* strategies applied to single name stocks, the diversification potential is significantly higher, and the 95% threshold suggests that portfolios with up to 86 positions could still benefit significantly from additional diversification. These insights are far less intuitive than in the case of passive long-only strategies, but highly relevant for every trend-following manager and investor alike.

For the global futures universe our method reconfirms the importance and benefits of highly diversified portfolios: while the 95% threshold for a risk parity futures portfolio indicates a sufficient portfolio size of around 80 positions, a generic trend-following approach can further benefit from additional diversification of up to 126 positions.

In the case of Quantica's trend-following approach, the diversification benefit could be extended to more than 200 positions. This result highlights the importance of striving to identify opportunities to add sufficiently liquid and operationally accessible and tradable markets to the QMF investment universe. Recently, QMF's investment universe has been expanded from 64 to 84 markets. Any further expansion is clearly desirable, but a careful evaluation of the additional cost of adding less liquid markets has to be taken into consideration.

Finally, for portfolios of CTA products, the lack of significant diversification potential leads to extremely low 95% diversification thresholds. Portfolios of 5-6 different CTA products seem to take almost full advantage of the diversification potential. Therefore, the role of portfolio selection plays a much bigger role, and more diversified portfolios can be beneficial for other reasons than pure diversification, e.g. increased capacity, or to reduce single manager risks.

In summary, we have developed a general method applicable to any investment strategy whereby purely from observing the variation of the Sharpe ratio as the number of constituents is varied, we can infer a threshold for the maximum number of constituents one might want to diversify into for pure diversification benefits.

So far, we have demonstrated that the diversification benefit of trend-following strategies is significantly higher compared to traditional long-only strategies. Further evaluation of these features and benefits in trend-following strategies is the topic of the last section of this note.

A deeper look into the diversification potential of trend-following strategies

In this section, we extend our analysis to the behavior of individual asset classes in the global futures universe in the context of trend-following returns. Table 3 summarizes the average cross-correlations of instruments within each sub-sector as well as the average cross-correlations between each asset class for the generic trend-following model applied to our universe of 84 global futures markets.

Commodities and Currencies have historically shown the lowest average cross-correlation between their constituent generic trend-following return streams. The diversification threshold $N_{95\%}$ defined in the previous section can also be used to assess the remaining diversification potential that can be exploited by further expanding the investment universe within each asset class. Given their low intra asset class cross-correlation, a portfolio would need a lot more commodities than bonds before reaching its diversification threshold (by almost a factor of 5, as the $N_{95\%}$ is reached for 95 commodities versus only 19 bond instruments).

	Average Cross-Correlation within asset class	Average Cross-Correlation against other asset classes	Average Sharpe ratio	95% diversification threshold
Equity	29%	12%	0.10	24
Bonds	33%	2%	0.56	19
FX	23%	19%	-0.01	33
Commodities	9%	20%	0.16	95

Table 3: Cross-correlation, Sharpe ratio and diversification threshold for different asset class generic trend-following return streams. Data covers the period 2010-2021. Source: Quantica Capital.

Zooming out further and looking at the cross-correlation between asset classes shows the diversification benefit of each sector on a stand-alone basis. As Table 3 also highlights, the lower the correlation of the return stream of an asset class against the others, the higher its potential diversification benefit.

As it becomes harder to find new uncorrelated and liquid asset classes to invest in, reducing the intra asset-class cross-correlation is a further step to improve the diversification benefit of a trend-following portfolio.

When selecting new instruments to add to an existing strategy, correlation metrics should be used to assess which instruments will add the most value to the portfolio from a pure diversification perspective. Without considering cost or operational constraints, preferences should be given to instruments that minimize the following three characteristics: average cross-correlation within its sector, average cross-correlation with all the other instruments and maximum cross-correlation with other instruments. The third metric is important as an instrument can display an average zero correlation within its sector and against the other instruments but still be extremely correlated to one single instrument already present in the strategy.

Table 4 displays the three correlation metrics that result from applying a generic trend-following strategy to the universe of liquid futures considered in our analysis. As mentioned above, currencies and commodities display the lowest concentration within their sector, with low cross-correlations and low maximum correlations reached per instrument.

However, somewhat counter-intuitively, Bonds and Equities add more diversification benefits than Commodities and FX on an asset class basis, as their average cross-asset class correlations are lower.

In summary, the methodology outlined for quantifying diversification benefits not only serves as a high-level analytical tool but, more importantly, can be used in both universe selection and portfolio construction to improve the Sharpe ratio of a trend-following strategy purely through diversification.

In the context of trend-following, which, as we have shown, benefits significantly from diversification due to its nature of lowly correlated return streams, the method can be applied at different levels of granularity to enhance and 'optimize' the investment process through pure diversification.

Generic Trend-Following (broad futures universe)	Asset Class	Average total cross-correlation	Average intra asset-class correlation	Maximum cross-correlation	Instrument most correlated with
S&P 500	Equities	11%	34%	91%	Dow
SMI (Swiss)	Equities	8%	28%	56%	OMXS30 (Sweden)
Nikkei	Equities	10%	26%	55%	FX JPY
Dow	Equities	9%	30%	91%	S&P 500
Euro StoXX	Equities	13%	35%	88%	CAC 40
SPI (Australia)	Equities	5%	16%	33%	Nikkei 225
S&P/TSX 60 (Can)	Equities	8%	26%	53%	S&P 500
FTSE 100 (UK)	Equities	12%	30%	60%	AEX (Netherlands)
FTSE Taiwan	Equities	5%	15%	24%	Nikkei 225
Russel 2000	Equities	10%	29%	89%	E-mini S&P MidCap 400
DAX	Equities	12%	35%	80%	Euro StoXX
Topix	Equities	6%	17%	92%	Nikkei 225
Nasdaq	Equities	9%	29%	83%	S&P 500
CAC 40	Equities	13%	35%	88%	Euro StoXX
Hang Seng	Equities	6%	17%	76%	HSCEI (HK)
AEX (Netherlands)	Equities	14%	37%	79%	CAC 40
OMXS30 (Sweden)	Equities	10%	34%	66%	DAX
HSCEI (HK)	Equities	5%	12%	76%	Hang Seng
MSCI EM	Equities	11%	19%	51%	MSCI EAFE
MSCI Singapore	Equities	7%	17%	36%	Hang Seng
SGX FTSE China A50 Index	Equities	3%	8%	40%	HSCEI (HK)
FTSE/MIB	Equities	11%	25%	72%	Euro StoXX
Nikkei 225	Equities	6%	18%	92%	Topix
MSCI EAFE	Equities	15%	36%	61%	E-mini S&P MidCap 400
Nifty SGX	Equities	6%	11%	26%	Hang Seng
E-mini S&P MidCap 400	Equities	10%	31%	89%	Russel 2000
USD Note 2yr	Fixed Income	7%	29%	84%	IR USD 3m
USD Note 5yr	Fixed Income	10%	40%	91%	USD Treasury 10yr
USD Treasury 10yr	Fixed Income	11%	46%	94%	Ultra 10-Year US Treasury Note
Ultra 10-Year US Treasury Note	Fixed Income	11%	46%	94%	USD Treasury 10yr
USD Long 20yr	Fixed Income	11%	47%	96%	USD Ultra 30yr
USD Ultra 30yr	Fixed Income	10%	43%	96%	USD Long 20yr
EUR Schatz 2yr	Fixed Income	6%	26%	78%	EUR Bobl 5yr
EUR Bobl 5yr	Fixed Income	9%	38%	83%	EUR Bund 10yr
EUR Bund 10yr	Fixed Income	10%	44%	90%	EUR Buxl 30yr
EUR Buxl 30yr	Fixed Income	9%	41%	90%	EUR Bund 10yr
Australian Gov 3Y	Fixed Income	5%	14%	77%	AUD Treasury 10yr
AUD Treasury 10yr	Fixed Income	5%	18%	77%	Australian Gov 3Y
Short term Euro-BTP	Fixed Income	5%	5%	73%	EUR BTP Italy 10yr
EUR BTP Italy 10yr	Fixed Income	6%	11%	73%	Short term Euro-BTP
JPY Bond 10yr	Fixed Income	2%	11%	28%	AUD Treasury 10yr
GBP Gilt 10yr	Fixed Income	11%	41%	66%	EUR Bund 10yr
CAD Treasury 10yr	Fixed Income	11%	42%	77%	USD Long 20yr
EUR OAT France 10yr	Fixed Income	8%	34%	76%	EUR Bund 10yr
IR USD 3m	Short Rates	7%	23%	84%	USD Note 2yr
IR GBP 3m	Short Rates	7%	25%	51%	GBP Gilt 10yr
IR AUD 3m	Short Rates	3%	6%	70%	Australian Gov 3Y
IR EUR 3m	Short Rates	7%	24%	71%	EUR Bobl 5yr
IR CAD 3m	Short Rates	4%	14%	34%	CAD Treasury 10yr
FX EUR	Currencies	6%	26%	43%	FX GBP
FX GBP	Currencies	5%	20%	43%	FX EUR
FX CHF	Currencies	4%	15%	37%	FX EUR
FX CAD	Currencies	8%	22%	45%	FX AUD
FX JPY	Currencies	6%	13%	55%	Nikkei
FX AUD	Currencies	7%	28%	60%	FX NZD
FX NZD	Currencies	5%	27%	60%	FX AUD
FX MXN	Currencies	7%	14%	31%	MSCI EM
Crude	Commodities	10%	19%	97%	WTI Crude Oil
Brent Crude	Commodities	11%	21%	86%	Heating Oil
WTI Crude Oil	Commodities	10%	19%	97%	Crude
Heating Oil	Commodities	10%	21%	86%	Brent Crude
Gasoline	Commodities	9%	17%	73%	Brent Crude
Gasoil LS	Commodities	8%	16%	69%	Heating Oil
Natural Gas	Commodities	0%	2%	8%	Heating Oil
CO2 Emissions	Commodities	2%	2%	8%	Gasoil LS
Gold	Commodities	7%	9%	66%	Silver
Copper	Commodities	7%	11%	31%	Platinum
Silver	Commodities	6%	11%	66%	Gold
Platinum	Commodities	8%	12%	40%	Silver
Nymex Palladium	Commodities	5%	7%	34%	Platinum
Iron Ore	Commodities	1%	2%	12%	Copper
Corn	Commodities	4%	11%	38%	Hard Red Winter Wheat
Wheat	Commodities	3%	7%	83%	Hard Red Winter Wheat future
Soy Beans	Commodities	3%	10%	71%	Soybean Meal
Hard Red Winter Wheat	Commodities	4%	9%	83%	Wheat
Live Cattle	Commodities	2%	3%	17%	Lean Hogs
Lean Hogs	Commodities	1%	2%	17%	Live Cattle
Sugar	Commodities	3%	7%	20%	Coffee
Coffee	Commodities	2%	5%	20%	Sugar
Soybean Meal	Commodities	1%	6%	71%	Soy Beans
Cotton	Commodities	6%	6%	16%	MSCI EAFE
Cocoa	Commodities	1%	1%	9%	FX GBP
Crude soybean oil	Commodities	4%	9%	33%	Soy Beans
Natural Gas Dutch TTF	Commodities	2%	5%	25%	CO2 Emissions

Table 4: Average and maximum cross-correlations within and across asset classes for a universe of 84 highly liquid futures instruments. Data covers the period 2010-2021. Source: Quantica Capital.

Conclusion

We have presented a framework to assess the benefits and quantify the limits of diversification for any investment portfolio.

Starting from a simple theoretical model, we have shown that the diversification gain when expanding a portfolio depends essentially only on the average cross-correlation of the underlying return streams and the number of portfolio constituents.

With a simple formula, we have demonstrated that the diversification potential is inversely proportional to the average cross-correlation of the underlying return streams. The non-linear relationship between a portfolio's underlying average cross-correlation and the resulting portfolio Sharpe ratio leads to significant differences for different investment strategies.

We have applied the theoretical model to different asset classes and investment strategies and have validated this model with a much more general simulation method and found very good empirical agreement with the model.

Our results prove that the diversification potential of trend-following strategies is significantly higher than e.g., for equity long-only or risk parity strategies. For pure stock portfolios, we have demonstrated that the diversification potential is no longer significant for portfolio sizes bigger than 15-20 positions, and the Sharpe ratio cannot be increased much more through pure diversification.

However, for trend-following strategies applied to a universe of liquid futures markets, this *diversification threshold* is significantly higher. Trend-following portfolios with 100 positions (for generic trend-following models) and even 200 positions (for Quantica's risk-adjusted, relative trend-following methodology) could still benefit from a sizable diversification benefit when considering a further investment universe expansion.

Although desirable, the hurdle in terms of higher complexity, implementation and operational costs will increase meaningfully the more futures markets are added to the investment universe.

Therefore, continuous monitoring of all available investment opportunities, and selection of potential candidates for inclusion in a trend-following strategy plays a crucial role and should be an important part of the research process of every trend-following manager.

Finally, a careful evaluation and integration of diversification and correlation considerations into the investment process can be crucial to the long-term success of a trend following strategy. By pure diversification, portfolio Sharpe ratio can be increased to a well quantifiable level, and without compromising style consistency or other desirable features of the trend-following strategy.

Appendix

Derivation of the diversification formula

For (log-)return streams $R_1, \dots, R_n$ with equal variances σ^2 and given cross-correlations $\rho_{ij} = Corr(R_i, R_j)$ we can express the variance of their average in terms of the average cross-correlation $\bar{\rho} = \frac{1}{n^2-n}\sum_{i\neq j}\rho_{ij}$:

$$\begin{aligned}\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} R_i\right) &= \frac{1}{n^2}\left(\sum_{i=1}^{n}\mathrm{Var}(R_i) + \sum_{i\neq j}\mathrm{Cov}(R_i, R_j)\right)\\ &= \frac{1}{n^2}\left(n\sigma^2 + \sum_{i\neq j}\sigma^2\rho_{ij}\right)\\ &= \frac{\sigma^2}{n^2}\left(n + (n^2-n)\bar{\rho}\right)\\ &= \sigma^2\left(\bar{\rho} + \frac{1-\bar{\rho}}{n}\right).\end{aligned}$$

Derivation of the Sharpe ratio of a portfolio

The Sharpe ratio of a portfolio of risky assets with equal volatilities can be expressed as a function of the average Sharpe ratio, the number of portfolio constituents and the average correlation:

$$\begin{aligned}\mathrm{Sharpe}\left(\frac{1}{n}\sum_{i=1}^{n} R_i\right) &= \frac{E\left[\frac{1}{n}\sum_{i=1}^{n} R_i\right]}{\sqrt{\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} R_i\right)}}\\ &= \frac{\frac{1}{n}\sum_{i=1}^{n} E[R_i]}{\sigma\cdot\sqrt{\bar{\rho} + \frac{1-\bar{\rho}}{n}}} = \bar{s}\cdot\frac{1}{\sqrt{\bar{\rho} + \frac{1-\bar{\rho}}{n}}}\\ &= \bar{s}\cdot m(\bar{\rho}, n)\,.\end{aligned}$$

Derivation of the diversification threshold

Recall that for a given Sharpe ratio and correlation characteristics $\bar{s}$ and $\bar{\rho} > 0$, the maximum achievable Sharpe ratio is $\bar{s}\cdot\frac{1}{\sqrt{\bar{\rho}}}$. Hence the smallest number of constituents N_p needed to reach a fraction $p \in [0,1]$ of this maximum will have to satisfy the inequality

$$p \leq \frac{\bar{s}\cdot m(\bar{\rho}, N_p)}{\bar{s}\cdot\frac{1}{\sqrt{\bar{\rho}}}} = \sqrt{\frac{\bar{\rho}}{\bar{\rho} + \frac{1-\bar{\rho}}{N_p}}}\,.$$

Squaring both sides of the inequality and assuming $\bar{\rho} > 0$ we can rewrite this as

$$N_p\bar{\rho}p^2 + p^2 - \bar{\rho}p^2 \leq N_p\bar{\rho}\,,$$

which is equivalent to

$$N_p(\bar{\rho} - \bar{\rho}p^2) \geq p^2 - \bar{\rho}p^2\,.$$

This in turn gives us the final desired inequality

$$N_p \geq \frac{(1-\bar{\rho})p^2}{\bar{\rho}(1-p^2)} = \frac{1-\bar{\rho}}{\bar{\rho}}\cdot\frac{p^2}{1-p^2}\,.$$

Since 2003, Quantica Capital's mission has been to design and implement the best possible systematic trend-following investment products in highly liquid, global markets. To the benefit of our investors and all our stakeholders.

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